Theory of machinery

Chapter five

Dynamics Forces Analysis

By

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Dynamics Forces Analysis

Static analysis

It is required to determine the input load due to a known output load , and it is necessary to determine the joints forces for the purpose of design and selection of mechanism components

Procedures

1- Draw free body diagram for each link

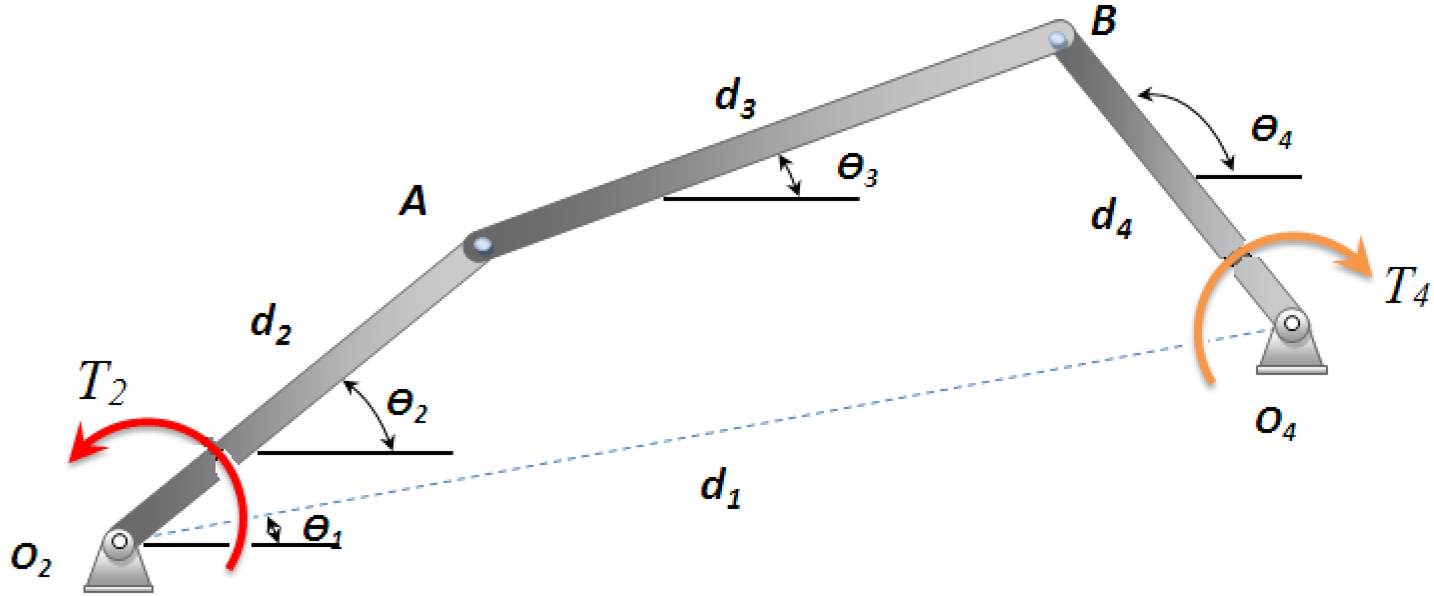
2- Write static equilibrium equations

3- Solve for the known reactions and driving loads

Dynamics Forces Analysis

4- bars Mech.

Given 4-bar shown T_2 is the driving torque and T_4 is the output load , find T_4 in terms of T_2 and the pin reactions

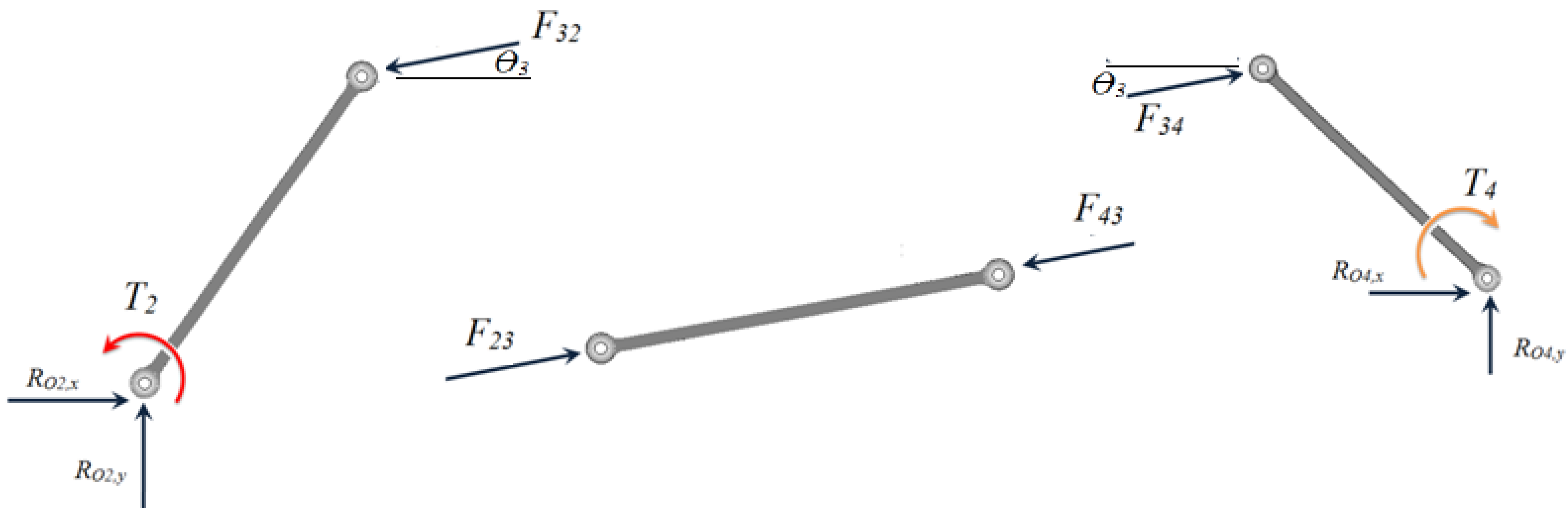


Dynamics Forces Analysis

T h e o r y o f m a c h i n e r y

4- bars Mech.

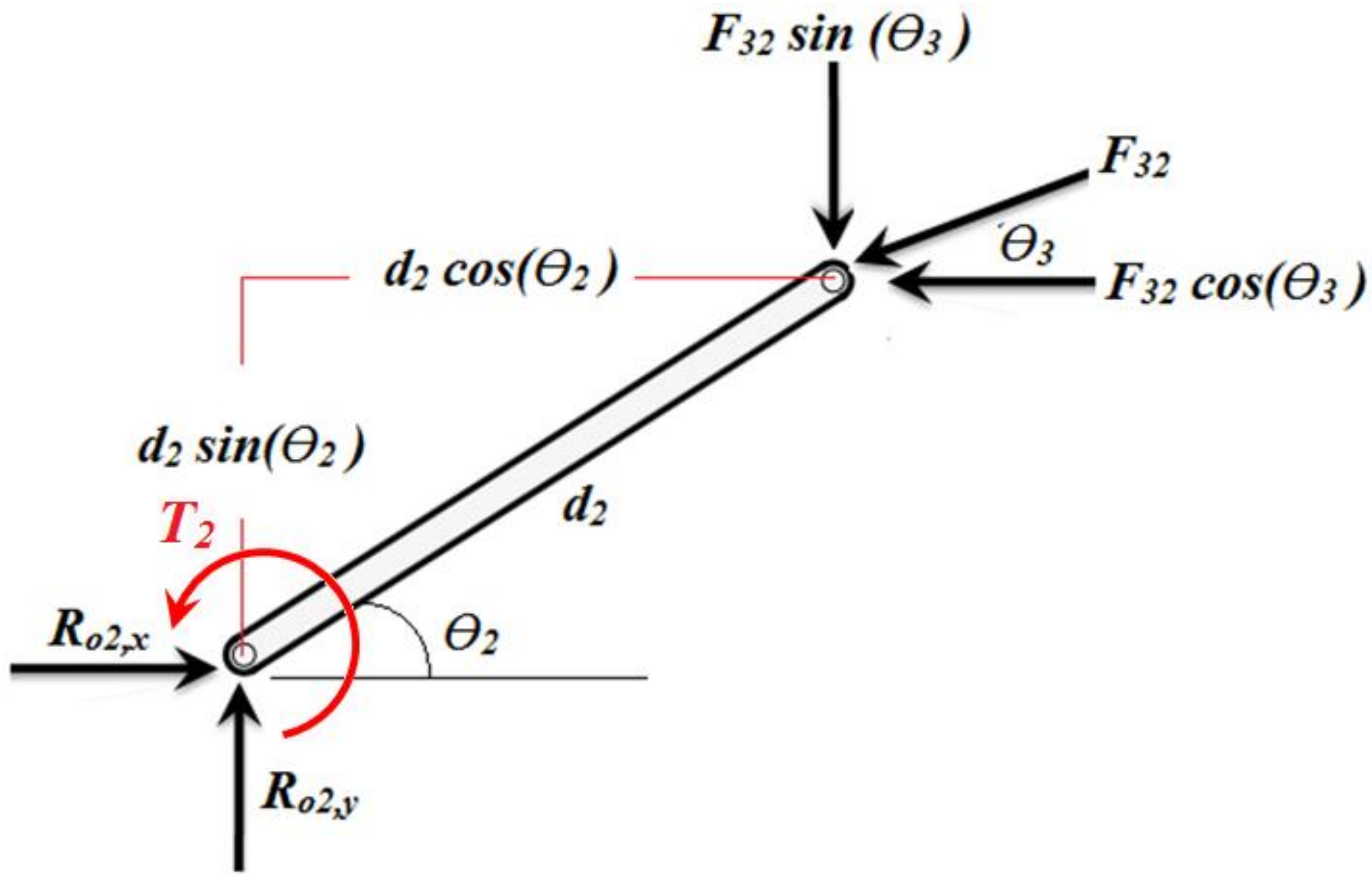
first the free body diagram for each link



Dynamics Forces Analysis

S.E.E (static equilibrium equations)

Link 2: F.B.D



Dynamics Forces Analysis

S.E.E (static equilibrium equations)

Link 2: Equilibrium equations

To find the reactions at point O_2 and force F_{32}, we can apply the equilibrium equations

$$\sum F_x = 0 \Rightarrow R_{O2,x} - F_{32}\cos(\theta_3) = 0 - - - (1)$$

$$\sum F_y = 0 \Rightarrow R_{O2,y} - F_{32}\sin(\theta_3) = 0 - - - (2)$$

$$\sum M_{O2} = 0 \Rightarrow T_2 + F_{32}\cos(\theta_3)(d_2)\sin(\theta_2) - F_{32}\sin(\theta_3)(d_2)\cos(\theta_2) = 0 - - - (3)$$

$$or\ T_2 = F_{32}\sin(\theta_3)(d_2)\cos(\theta_2) - F_{32}\cos(\theta_3)(d_2)\sin(\theta_2)$$

But, $\sin(\Theta_3)\cos(\Theta_2) - \cos(\Theta_3)\sin(\Theta_2) = \sin(\Theta_3 - \Theta_2)$. So, T_2 can be found as:

$$T_2 = F_{32} d_2 \sin(\theta_3 - \theta_2)$$

Dynamics Forces Analysis

S.E.E (static equilibrium equations)

Link 3:

Link 3 is considered as two forces member since it carry the force from the input link to the output link
The equilibrium equation for such case is shown in Eq.4:

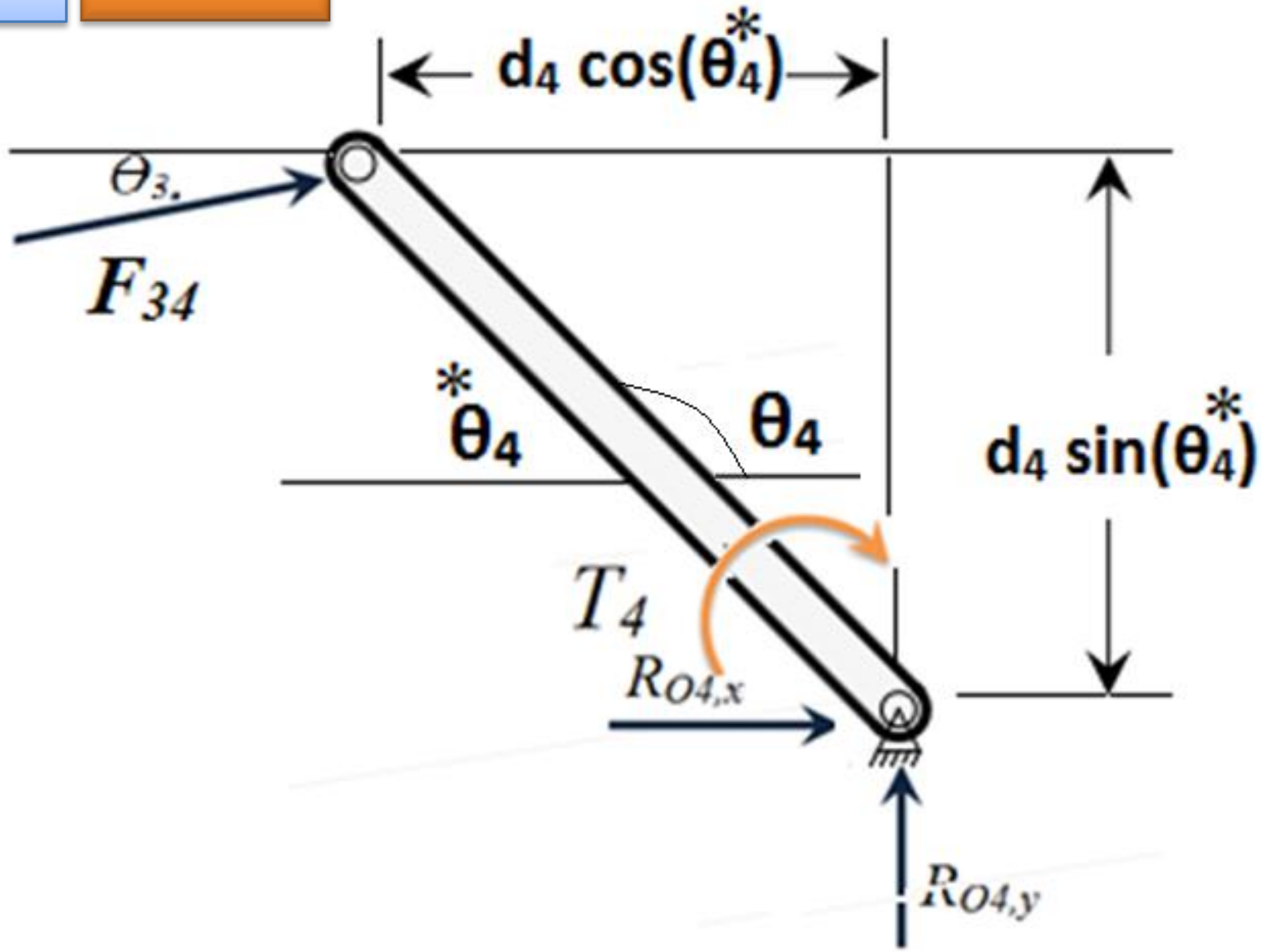
$$F_{43} = F_{23} - - - (4)$$

T h e o r y o f m a c h i n e r y

Dynamics Forces Analysis

S.E.E (static equilibrium equations)

Link 4: F.B.D



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Dynamics Forces Analysis

Link 4: **Equilibrium equations**

Before starting the solution, a Trigonometrical Ratios must be mentioned which couple ϴ with 180 – ϴ:

sin (180 – ϴ) = ***sin (ϴ)*** and ***cos(180 – ϴ)*** = ***– cos (ϴ)***

And because *ϴ*$_4$ = 180 – ϴ$_4$*, *sin(ϴ*$_4$) = sin(ϴ$_4$)* and *cos(ϴ*$_4$) = – cos(ϴ$_4$)*

Now, the equilibrium equations can be found as:

$$\sum F_x = 0 \Rightarrow R_{O4,x} + F_{34}\cos(\theta_3) = 0 --- (5)$$

$$\sum F_y = 0 \Rightarrow R_{O4,y} - F_{34}\sin(\theta_3) = 0 --- (6)$$

= -cos(ϴ$_4$)

$$\sum M_{O2} = 0 \Rightarrow -T_4 - F_{34}d_4\cos(\theta_3)\sin(\theta_4^*) - F_{34}d_4\sin(\theta_3)\cos(\theta_4^*) = 0$$

$$\Rightarrow -T_4 + F_{34}d_4[\sin(\theta_3)\cos(\theta_4) - \cos(\theta_3)\sin(\theta_4)] = 0 --- (7)$$

But, sin(ϴ$_3$) cos(ϴ$_4$) – cos(ϴ$_3$) sin(ϴ$_4$) = sin(ϴ$_3$ - ϴ$_4$). So, T_2 can be found as:

$$T_4 = F_{32}d_4\sin(\theta_3 - \theta_4)$$

Dynamics Forces Analysis

S.E.E (static equilibrium equations)

Finally

Relates the torques founded from Eqs 3 and 7

$$T_2 = F_{32} d_2 \sin(\theta_3 - \theta_2) \qquad T_4 = F_{32} d_4 \sin(\theta_3 - \theta_4)$$

$$\frac{T_4}{T_2} = \frac{F_{32} d_4 \sin(\theta_3 - \theta_4)}{F_{32} d_2 \sin(\theta_3 - \theta_2)} = \frac{d_4 \sin(\theta_3 - \theta_4)}{d_2 \sin(\theta_3 - \theta_2)} = \frac{\omega_2}{\omega_4}$$

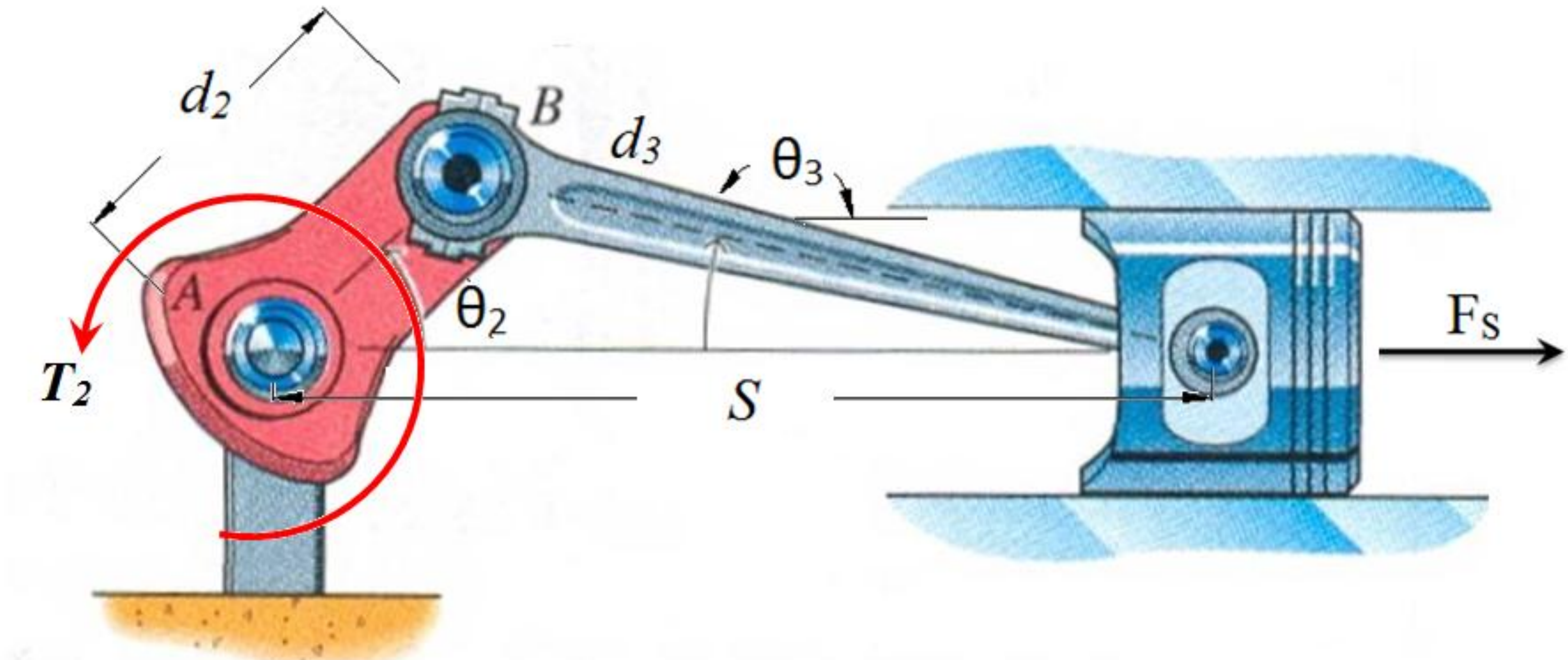
The previous conclusion between the torques ratio and velocities ratio can be proofed if we assume there are no losses among the power transition between the input and output links (i.e. $P_2 = P_4$).

If $P_2 = P_4$, then $T_2\omega_2 = T_4\omega_4$ **or** $T_4/T_2 = \omega_2/\omega_4$.

Dynamics Forces Analysis

Slider crank mechanism

- Input T_2 , relate T_2 to Fs and find the pin reactions



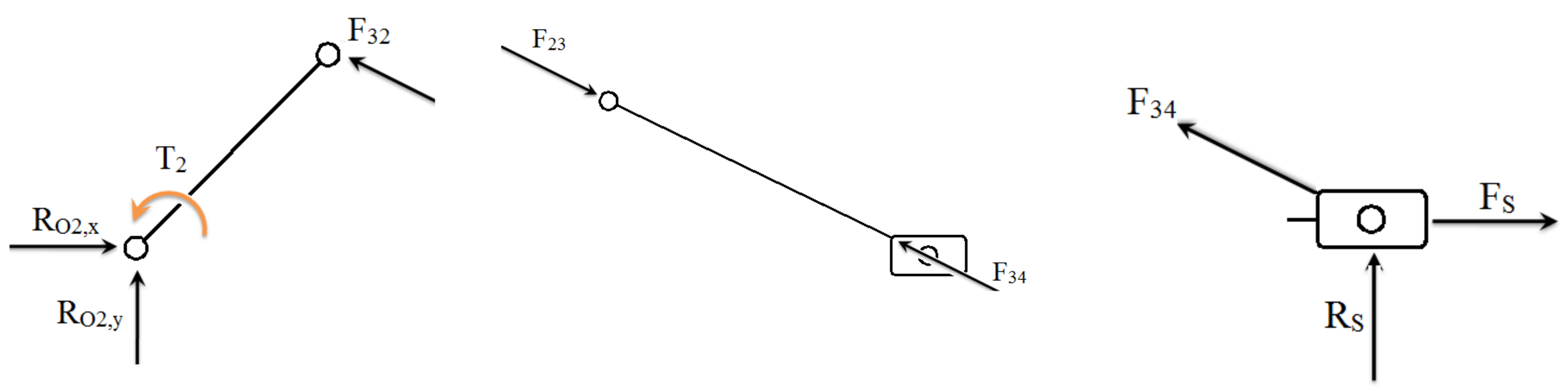
Dynamics Forces Analysis

Slider crank mechansim

- Input T_2 is input, relate T_2 to Fs and find the pin reactions
- Link 2: $\sum F_x = R_{o2x} - F_{32}\cos(\theta_3) = 0$ ----(1) ; $\sum F_y = R_{o2y} - F_{32}\sin(\theta_3) = 0$ ---- (2)

  $T_2 = -\ F_{32}\, d_2 \sin(\theta_3 - \theta_2)$ ---- (3)
- Link 3: $F_{43} = F_{23} = F_{32}$---- (4)
- Link 4: $\sum F_x = F_s - F_{34}\cos(\theta_3) = 0$ ---- (5)

  $\sum F_y = F_{34}\sin(\theta_3) + Rs = 0$ ---- (6)
- We have 6 unknown with 6 equations solve them to find the unknown (F_s, F_{32}, R_{o2x}, R_{o2y} ,F_{34}, R_s)



Theory of machinery

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Dynamics Forces Analysis

Dynamic analysis

It is required to find the dynamic reactions at the joints and the driving forces or torques for given mechanism parameters and loads

Given :
1- configuration
2- dimensions
3- location of fixed pivots and plats
4- inertial parameters
5- loads

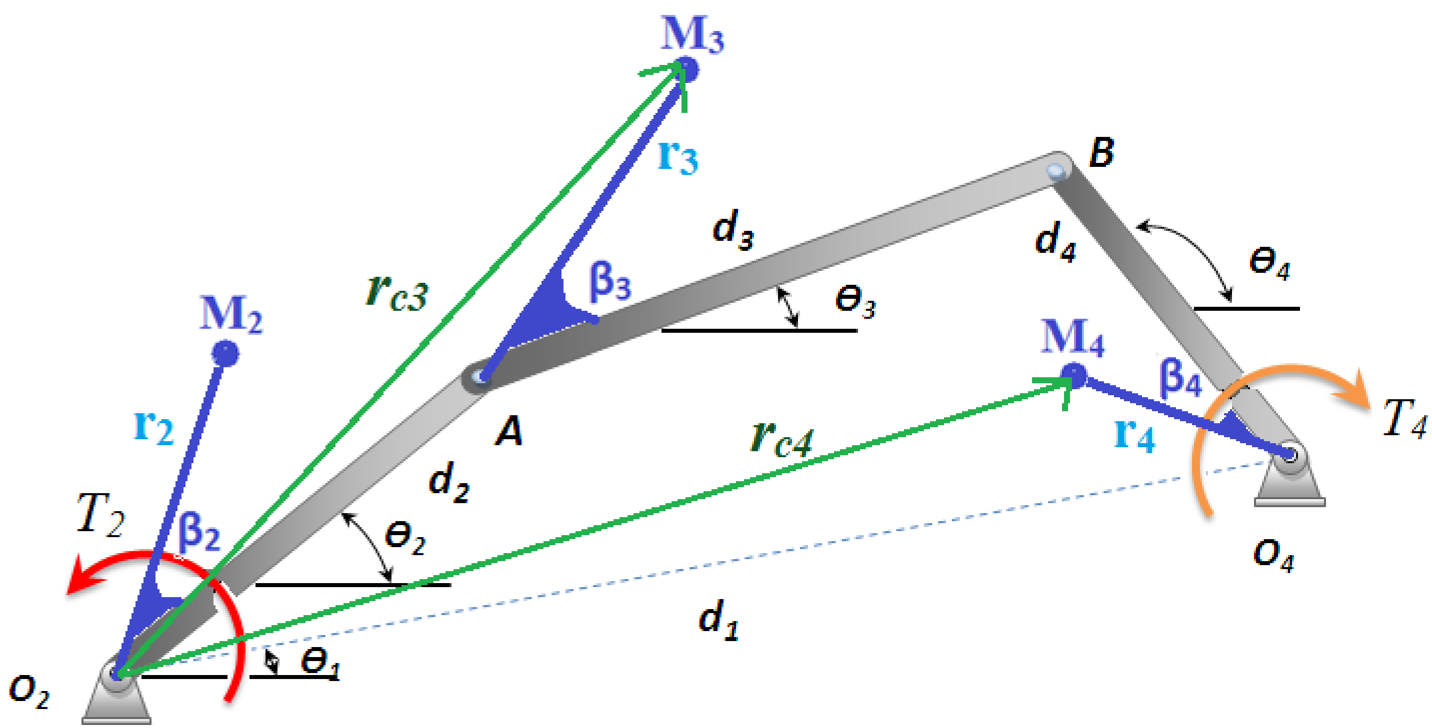
Procedures :
1- Free body diagram with inertial loads ($\sum F = ma$)
2- Write the dynamic equilibrium equations
3- Solve for the unknown forces and reactions

T h e o r y o f m a c h i n e r y

Dynamics Forces Analysis

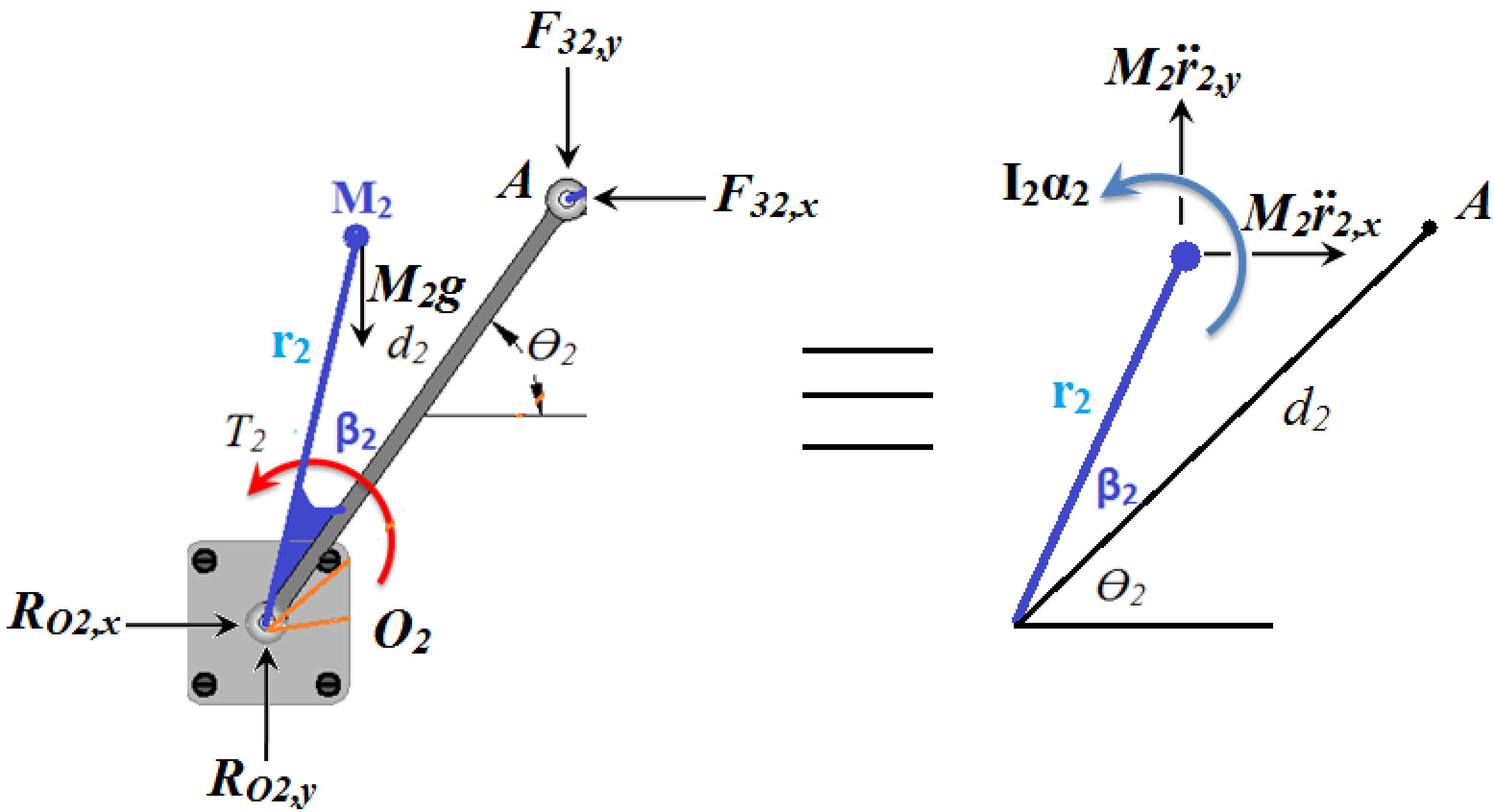
Dynamic analysis

4-bar mechanism



Dynamics Forces Analysis

link 2:



Dynamics Forces Analysis

Theory of machinery

link 2:

$$\sum F_x = 0 \Rightarrow R_{O2,x} - F_{32,x} = M_2 \ddot{r}_{2x} ---(1)$$

$$\sum F_y = 0 \Rightarrow R_{O2,y} - F_{32,y} - M_2 g = M_2 \ddot{r}_{2y} ---(2)$$

$$\sum M_{O2} = T_2 + F_{32,x}(d_2)\sin(\theta_2) - F_{32,y}(d_2)\cos(\theta_2) - m_2 g r_2 \cos(\theta_2 + \beta_2)$$

$$= I_2\alpha_2 - M_2 \ddot{r}_{2x} r_2 \sin(\theta_2 + \beta_2) + M_2 \ddot{r}_{2y} r_2 \cos(\theta_2 + \beta_2) ---(3)$$

$$\mathbf{r_2} = r_2 U_{\theta_2+\beta_2}$$

$$\dot{\mathbf{r}}_\mathbf{2} = r_2\omega_2 \dot{U}_{\theta_2+\beta_2}$$

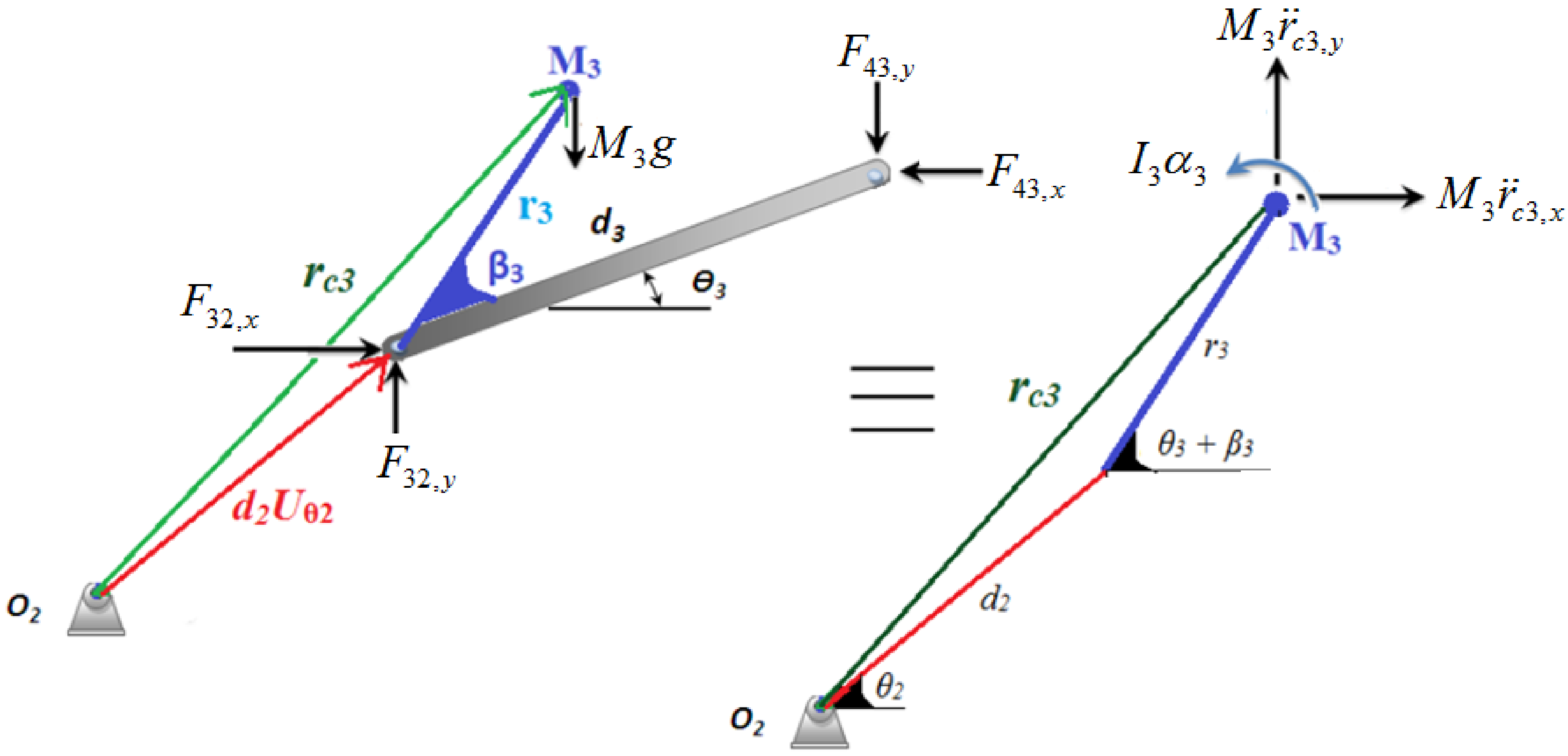
$$\ddot{\mathbf{r}}_\mathbf{2} = r_2\alpha_2 \dot{U}_{\theta_2+\beta_2} - r_2\omega_2^2 U_{\theta_2+\beta_2}$$

$$\ddot{\mathbf{r}}_\mathbf{2,x} = -r_2\alpha_2 \sin(\theta_2 + \beta_2) - r_2\omega_2^2 \cos(\theta_2 + \beta_2)$$

$$\ddot{\mathbf{r}}_\mathbf{2,y} = r_2\alpha_2 \cos(\theta_2 + \beta_2) - r_2\omega_2^2 \sin(\theta_2 + \beta_2)$$

Dynamics Forces Analysis

link 3:

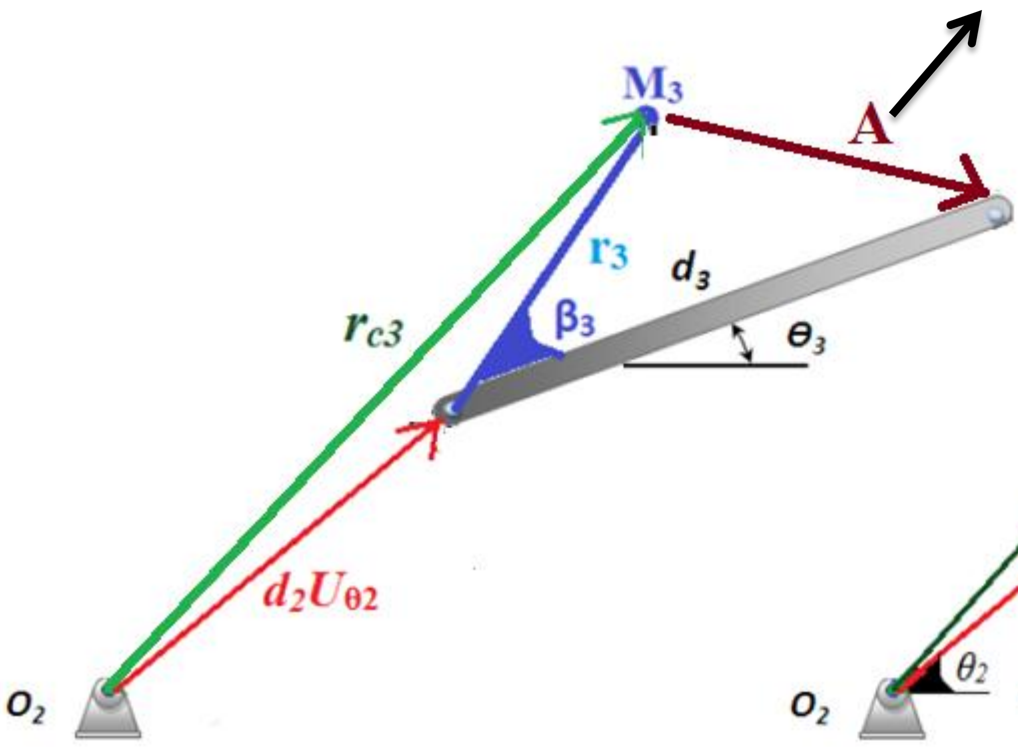


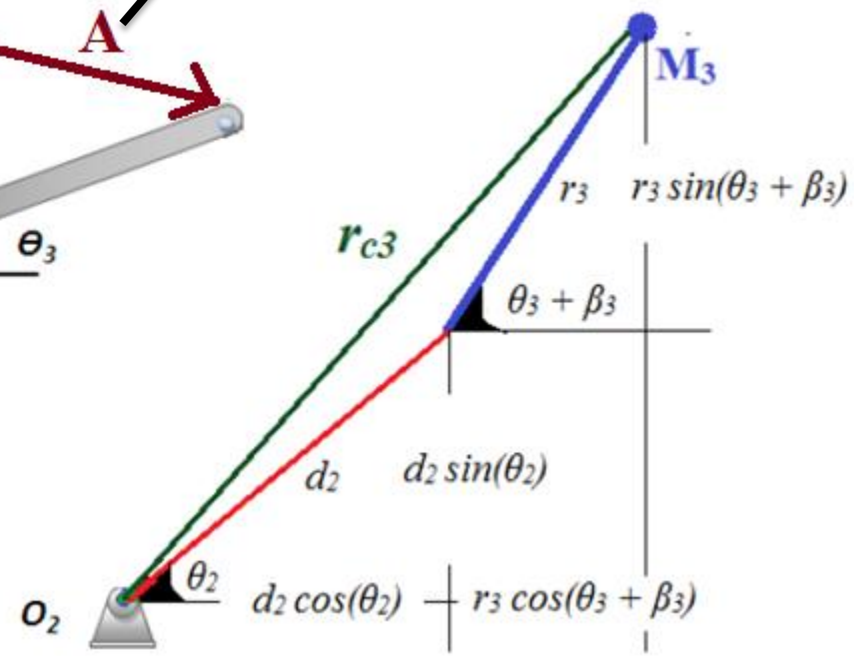
Dynamics Forces Analysis

Theory of machinery

link 3:

$$\vec{A} = \vec{d}_3 - \vec{r}_3$$





Dynamics Forces Analysis

T h e o r y o f m a c h i n e r y

link 3:

$$F_{32,x} - F_{43,x} = M_3 \ddot{r}_{c3,x} ---(4)$$

$$F_{32,y} - F_{43,y} - M_3 g = M_3 \ddot{r}_{c3,y} ---(5)$$

$$\sum M_B = F_{23,x}(d_3)\sin(\theta_3) - F_{23,y}(d_3)\cos(\theta_3) + M_3 g(d_3\cos(\theta_3) - r_3\cos(\theta_3+\beta_3))$$
$$= I_3\alpha_3 + M_3\ddot{r}_{3c,x}[d_3\sin(\theta_3) - r_3\sin(\theta_2+\beta_2)]$$
$$- M_3\ddot{r}_{3c,y}[d_3\cos(\theta_3) - r_3\cos(\theta_3+\beta_3)] ---(6)$$

$$\overrightarrow{r_{c3}} = d_2 U_{\theta_2} + r_3 U_{\theta_3+\beta_3}$$

Exercise:- Find the dynamic analysis for link 4